Word Problem #1 (Normal Distribution)

Suppose that the distribution of diastolic blood pressure in a population of hypertensive women is modeled well by a normal probability distribution with mean 100 mm Hg and standard deviation 14 mm Hg. Let X be the random variable representing this distribution. Find two symmetric values "a" and "b" such that

$$\text{Probability}\,[\, a < X < b \,] = .99$$

Word Problem #1 (Normal Distribution) – SOLUTION

Answer: a=63.95 b=136.05

Easy (but not as thoughtful) Solution:

Step 1

Launch the David Lane normal distribution calculator provided to you on the topic page (5. Normal) of the course website: **http://davidmlane.com/hyperstat/Z_table.html**

Step 2

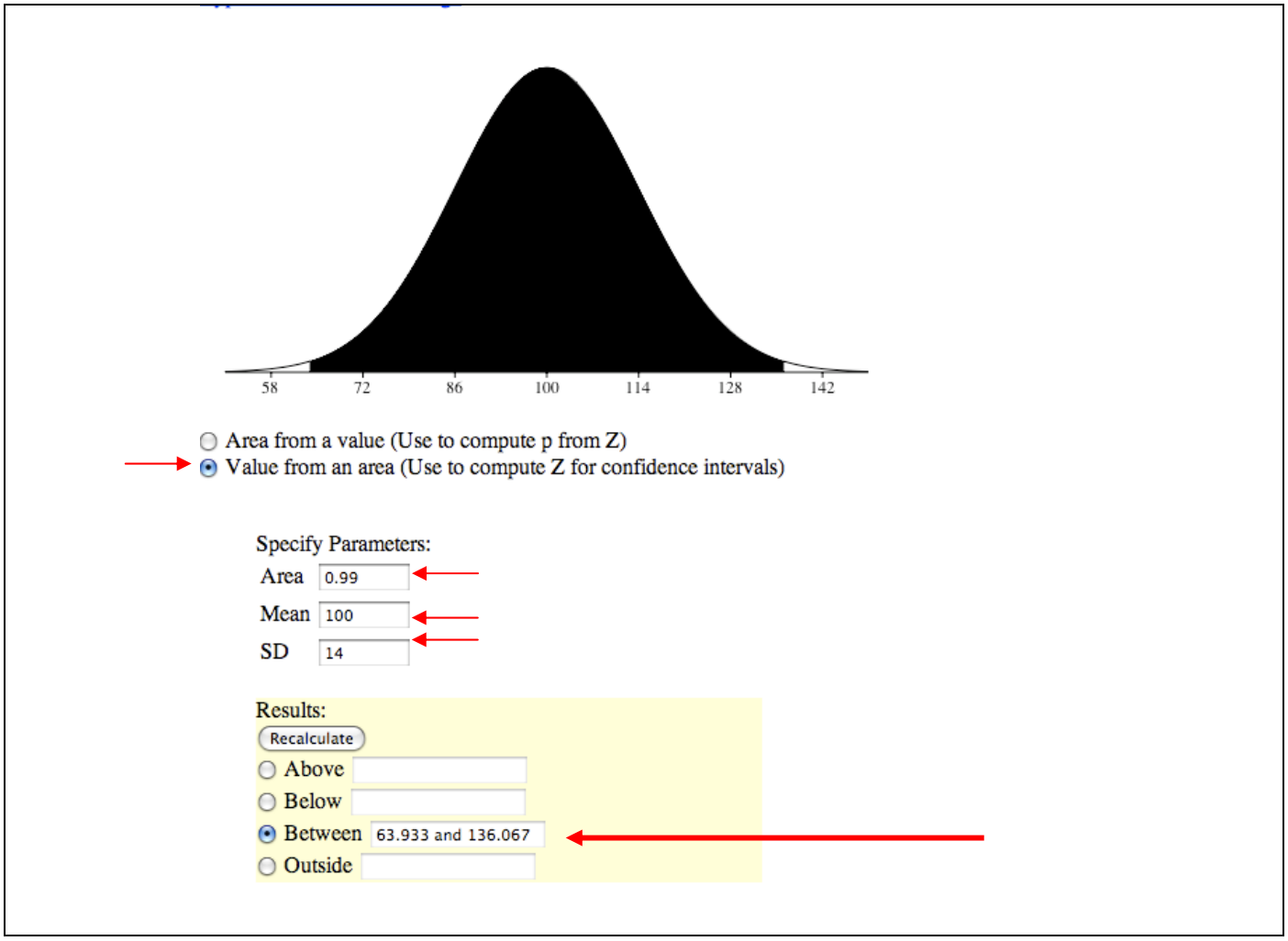
Click **on** the radio button to select, **"Value from an area (Use to compute Z for confidence intervals)"**

Step 3

In the box, labeled **area**, enter the value **.99,** in the box labeled **mean**, enter **100**, in the box labeled **SD** enter **14.**

Step 4

Click **on** the radio button to select, **"Between"**



Solution Using Z-Score:

Step 1

Launch the David Lane normal distribution calculator provided to you on the topic page (5. Normal) of the course website: **http://davidmlane.com/hyperstat/Z_table.html**

Step 2

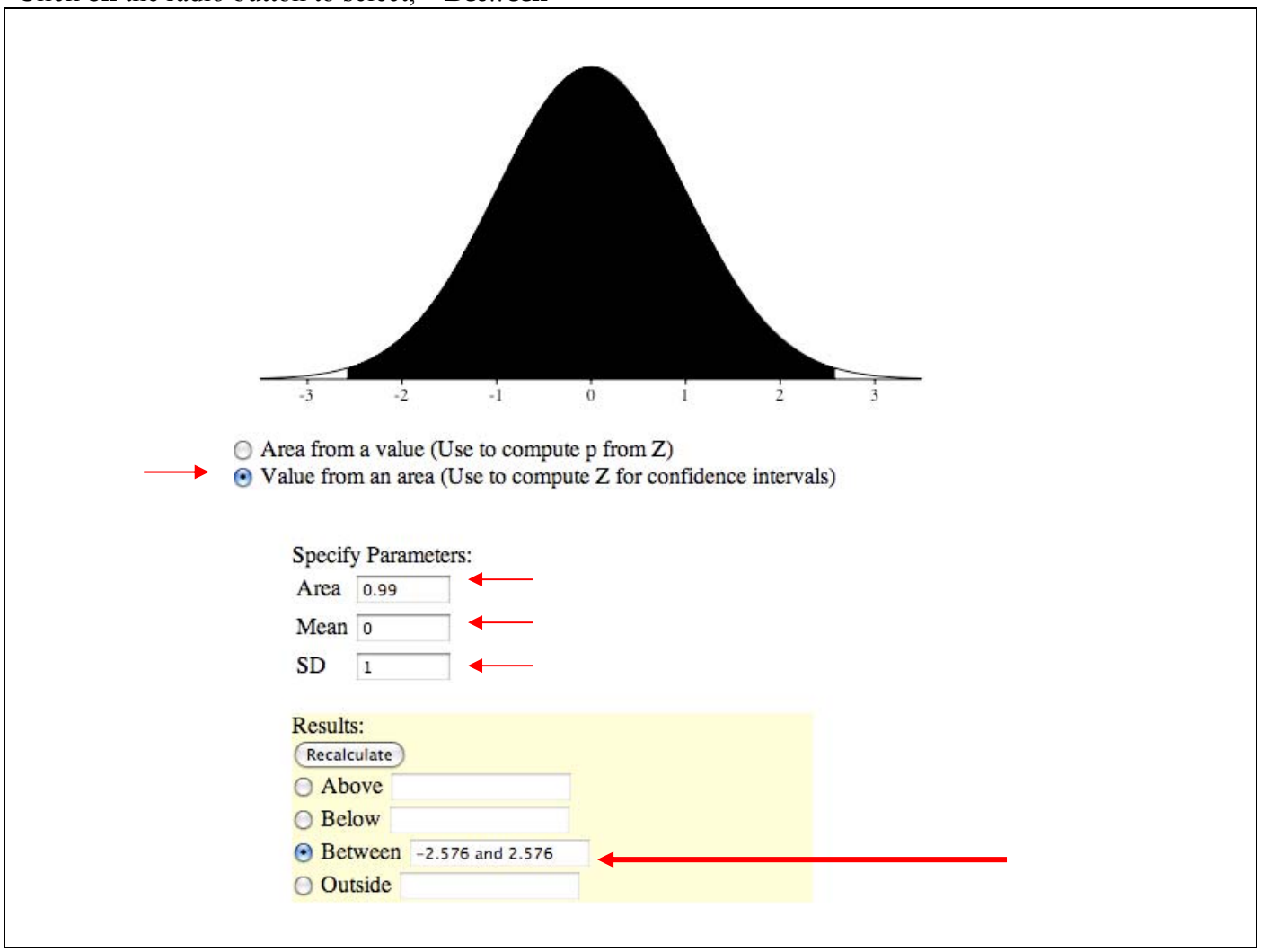
Click **on** the radio button to select, **"Value from an area (Use to compute Z for confidence intervals)"**

Step 3

In the box, labeled **area**, enter the value **.99,** in the box labeled **mean**, enter **0**, in the box labeled **SD** enter **1**.

Step 4

Click **on** the radio button to select, **"Between"**



Step 5

From the 0.5^{th} and 99.5^{th} percentiles of the standard normal distribution, solve for the corresponding values of the normal distribution that has mean=100 and sd=14.

Tip - Notice that the 0.5^{th} and 99.5^{th} percentiles are -2.57 and +2.57, symmetric about zero. So, really, we only needed to solve for one of them.

$$z = \frac{x-\mu}{\sigma} \text{ says that } \quad x=\sigma[z] + \mu$$

Thus a = 0.5th percentile for X = 14[-2.57] + 100 = 63.95
and b = 99.5th percentile for X = 14[+2.57] + 100 = 136.05

Word Problem #2 (Normal Distribution)

Suppose that the distribution of weights of New Zealand hamsters is distributed normal with mean 63.5 g and standard deviation 12.2 g. If there are 1000 weights in this population, how many of them are 78 g or greater?

Word Problem #2 (Normal Distribution) - SOLUTION
Answer: 117

Solution:

$$\Pr[\text{weight} > 78\text{ g}] = \Pr[\text{Normal } \mu=63.5\ \ \sigma=12.2\ > 78]$$

$$= \Pr\left[\text{Standard normal} > \frac{78-\mu}{\sigma}\right] = \Pr\left[\text{Standard normal} > \frac{78-63.5}{12.2}\right]$$

$$= \Pr[\text{Normal } (0,1)\ > 1.1885]$$

$$= .117$$

Therefore # Hamsters with weights > 78 g in a population of size 1000

= (1000)(.117)
= 117

Word Problem #3 (Normal Distribution)

Consider again the normal probability distribution of problem #2. What is the probability of selecting at random a sample of 10 hamsters that has a mean greater than 65 g?

Word Problem #3 (Normal Distribution) - SOLUTION
Answer: .3483

Easy Solution:

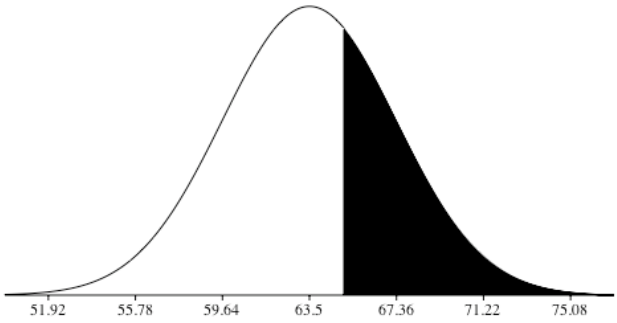
The solution to this problem requires noticing that the random variable is $\bar{X}$, so that the standardization to Z must use the SE of $\bar{X} = \sigma / \sqrt{n}$. ***Tip*** - But the David Lane calculator does not have a box for you labeled SE. It has only the box labeled SD. This is okay, however. Simply provide the value of the SE in the SD box.

Step 1
Solve for the value of the standard error of the sample mean. SE $= \sigma / \sqrt{n} =$ $12.2 / \sqrt{10} = 12.2 / 3.16 =$ **3.86**

Step 2
Click **on** the radio button to select, **"Area from a value (Use to compute p from Z)"**

Step 3
In the box, labeled **mean**, enter **63.5**, in the box labeled **SD** enter **3.86.**

Step 4
Click **on** the radio button to select, **"Above"** In the box at right, enter **65.** Click **recalculate**



Area from a value (Use to compute p from Z)
Value from an area (Use to compute Z for confidence intervals)

Specify Parameters:
Mean 63.5
SD 3.86
Above 65
Below 1.96
Between -1.96 and 1.96
Outside -1.96 and 1.96

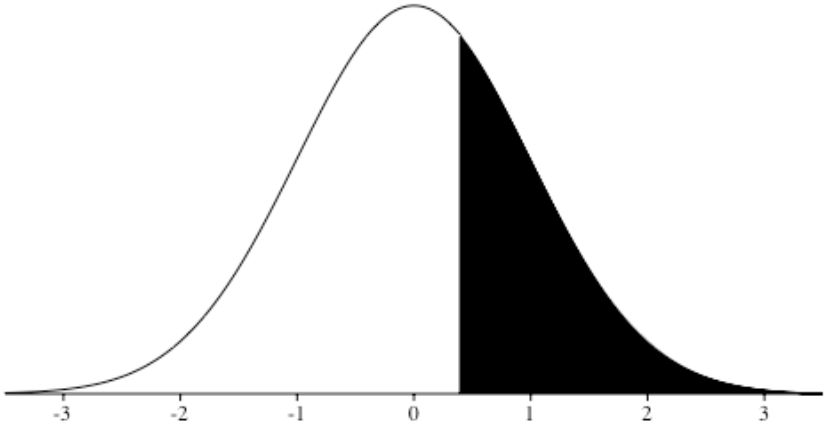
Results:
Area (probability) 0.3488
Recalculate

Solution Using Z-Score:

$$\Pr[\ \bar{X}_{n=10} > 65\ g\ ] = \Pr[\ \text{Normal}\ \mu_{\bar{X}}=63.5\ \ \sigma_{\bar{X}}=\frac{12.2}{\sqrt{10}}\ > 65\ ]$$

$$= \Pr[\ \text{Standard normal}\ > \frac{65-\mu_{\bar{X}}}{\sigma_{\bar{X}}}\ ] = \Pr[\ \text{Standard normal}\ > \frac{65-63.5}{12.2/\sqrt{10}}\ ]$$

$$= \Pr[\ \text{Normal}\ (0,1)\ > 0.3888\ ] = .3483$$



- (•) Area from a value (Use to compute p from Z)
- ( ) Value from an area (Use to compute Z for confidence intervals)

Specify Parameters:

Mean 0

SD 1

- (•) Above 0.3888
- ( ) Below 1.96
- ( ) Between -1.96 and 1.96
- ( ) Outside -1.96 and 1.96

Results:

Area (probability) 0.3487

Recalculate